

On Measurements, Numbers and p -Adic Mathematical Physics

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Abstract

In this short paper I consider relation between measurements, numbers and p -adic mathematical physics. p -Adic numbers are not result of measurements, but nevertheless they play significant role in description of some systems and phenomena. We illustrate their ability for applications referring to some sectors of p -adic mathematical physics and related topics, in particular, to string theory and the genetic code.

Dedicated to Igor V. Volovich on the occasion of his 65th birthday

1 Introduction

It is well known that science is based on experiments and their theoretical modelling. To my knowledge, Igor Volovich was the first mathematical physicist who pointed out the field \mathbb{Q} of rational numbers as a bridge between experiments and their possible theoretical descriptions [1]. Namely, the results of experimental measurements are some rational numbers, presented by finite number of digits. From mathematical point of view, \mathbb{Q} is dense in the field \mathbb{R} of real numbers and the fields \mathbb{Q}_p of p -adic numbers, for all primes p . On \mathbb{R} and \mathbb{Q}_p , as well as on their algebraic extensions, there are well developed analysis. Analysis on \mathbb{R} , and the field \mathbb{C} of complex numbers, is well employed in description of physical systems. In 1987 I. Volovich proposed [2] to use p -adic numbers in description of space-time at the Planck scale and in string theory, and he introduced concept of p -adic strings. It was first valuable application of p -adic analysis in mathematical physics and was marked beginning of

p -adic mathematical physics (for reviews see [3, 4, 5]). I am very glad for opportunity to collaborate with I. Volovich on application of p -adic numbers and adeles in various sectors of p -adic mathematical physics from its very beginning.

In the sequel of this paper I shall discuss some aspects of measurements, p -adic numbers and adeles, and their role in p -adic mathematical physics and related topics. In particular, I want to point out that results of measurements are rational numbers endowed by Archimedean norm and natural ordering. Also I want to emphasize that p -adic numbers, although being not direct result of measurements, are very important tools in description of some systems and phenomena. I shall demonstrate it at two examples – string theory and the genetic code.

2 Measurements, Numbers and Their Applicability

Any measurement is comparison of two quantities of the same kind, one quantity by convention is taken to be unit of measurement and the other one is subject of measuring. Comparing these two quantities we get their ratio, which contains some information how many times measured quantity is larger or smaller than the unit quantity. This ratio is a rational number, with only some finite number of certain digits. More precise measuring usually results in more certain digits. However, due to many reasons, there is always some uncertainty and result of measurement has only an approximative character. It is worth noting that these digits of the obtained rational number are digits in the decimal expansion of the real number. Hence, results of measurements are rational numbers with Archimedean norm and natural ordering, which are characteristics of real but not of p -adic numbers. Although this assertion is evident, it is not generally recognized.

Recall that, from algebraic point of view, \mathbb{Q} is a field and any its element x can be presented in the form $x = \frac{m}{n}$, where $m \in \mathbb{Z}$ and $n \in \mathbb{N}$. It is well known that summation and multiplication, and their inverse operations, are well defined for these numbers. Among rational numbers there is also another property, which is distance and is related to the norm. According to the Ostrowski theorem [6] there are no other non-trivial norms on \mathbb{Q} , which are non-equivalent either to the absolute value (Archimedean) or p -adic (non-Archimedean) norm, related to prime numbers. A rational number $x = \frac{m}{n} = p^\nu \frac{a}{b}$, where integers a and $b \neq 0$ are not divisible by prime number p , by definition has p -adic norm $|x|_p = p^{-\nu}$ and $|0|_p = 0$. Since $|x + y|_p \leq \max\{|x|_p, |y|_p\}$, p -adic norm is a non-Archimedean (ultrametric) one. As

completion of \mathbb{Q} with respect to the absolute value $|\cdot|_\infty$ gives the field $\mathbb{Q}_\infty \equiv \mathbb{R}$ of real numbers, by the same procedure using p -adic norm $|\cdot|_p$ one gets the field \mathbb{Q}_p of p -adic numbers (for any prime number $p = 2, 3, 5, \dots$). Any number $0 \neq x \in \mathbb{Q}_p$ has its unique canonical representation (see, e. g. [4])

$$x = p^\nu \sum_{n=0}^{+\infty} x_n p^n, \quad \nu \in \mathbb{Z}, \quad x_n \in \{0, 1, \dots, p-1\}, \quad x_0 \neq 0. \quad (1)$$

From representation (1) one can conclude that rational number obtained in the process of measuring cannot have this form with p -adic (non-Archimedean) norm. Also, p -adic numbers have not natural ordering, while results of measuring have it.

Measurements often are not direct, but with help of some tools and can be viewed as measuring of a length. By this way measurement is related to the Archimedean axiom in geometry, which is originally formulated for two segments on a straight line and it states: a larger segment (A) can be always surpassed by some finite number (n) of the successive addition of a smaller segment (a) along the larger one. This can be also expressed in terms of two real numbers: Let $a, A \in \mathbb{R}$ and $0 < |a|_\infty < |A|_\infty$ then there is always $n \in \mathbb{N}$ such that $|n|_\infty |a|_\infty > |A|_\infty$. Note that addition of a smaller segment along the larger one is just as measuring the larger segment by the smaller one. In the p -adic case, Archimedean axiom is not valid, because $|n|_p \leq 1$. Recall that one cannot measure distances smaller than the Planck length $\ell_P = \sqrt{\frac{\hbar G}{c^3}} \sim 10^{-35} m$, because quantum and gravity effects lead to the uncertainty which cannot be smaller than the Planck length ℓ_P . This was motivation that I. Volovich conjectured [1, 2] existence of non-Archimedean geometry with p -adic numbers at the Planck scale.

From achievements of applied mathematics it follows that systems of real numbers which have field structure, i.e. \mathbb{R} and \mathbb{C} , have the most applicability. For example, classical and quantum theoretical physics are mainly based on analysis related to maps $\mathbb{R} \rightarrow \mathbb{R}$ and $\mathbb{R} \rightarrow \mathbb{C}$, respectively. Comparing to \mathbb{R} and \mathbb{C} , normed division algebra of quaternions \mathbf{H} , which is noncommutative, is rather less applicative. Algebra of octonions \mathbf{O} as the last normed division algebra, which is noncommutative and nonassociative, has a minor role in applications. It is worth noting that complex numbers, which are not result of direct measuring and are unordered, are unavoidable in quantum mechanics, where they are used not for description of space-time but for complex-valued wave functions, which contain all information about the state of the quantum system.

Now one can pose the following question: Being not results of measurements, what role p -adic numbers can play in description of physical or some other systems?

p -Adic numbers should play unavoidable role where description with real numbers, or with systems of real numbers, is inadequate. In physical case such situation should be at the Planck scale, because it is not possible to measure distances smaller than the Planck length. It should be also the case with very complex physical, living, cognitive, information and some other complex systems and phenomena. For example, such systems may have space of states with some p -adic structure, like it is the case with quantum states described by complex-valued wave functions. Thus, we expect inevitability of p -adic numbers at some more profound levels in understanding of the Universe in its parts as well as a whole [7, 8, 9]. The first step towards possible p -adic level of knowledge is invention of relevant mathematical tools and construction of the adequate physical models. This is subject of p -adic mathematical physics and its brief recent overview is presented in [5] (see also [10]).

3 p -Adic Numbers in Applications

p -Adic numbers are discovered by Kurt Hansel in 1897 as a new tool in number theory. They have applications in many parts of modern mathematics: number theory, algebraic geometry, theory of representations, ... \mathbb{Q}_p is locally compact, complete and totally disconnected topological space. There is rich structure of algebraic extensions of \mathbb{Q}_p .

There are many possibilities for mappings between \mathbb{Q}_p . The most elaborated is analysis related to mappings $\mathbb{Q}_p \rightarrow \mathbb{Q}_p$ and $\mathbb{Q}_p \rightarrow \mathbb{C}$. Usual complex valued functions of p -adic argument are additive $\chi_p(x) = e^{2\pi i \{x\}_p}$ and multiplicative $|x|^s$ characters, where $\{x\}_p$ is fractional part of x and $s \in \mathbb{C}$ (for many aspects of p -adic numbers and their analysis, we refer to [3, 4, 6, 11]).

Real and p -adic numbers, as completions of \mathbb{Q} , are joined into adeles (see, e. g. [3, 4, 11, 12]). An adele α is an infinite sequence made of real and p -adic numbers in the form

$$\alpha = (\alpha_\infty, \alpha_2, \alpha_3, \dots, \alpha_p, \dots), \quad \alpha_\infty \in \mathbb{R}, \quad \alpha_p \in \mathbb{Q}_p, \quad (2)$$

where for all but a finite set \mathcal{P} of primes p it has to be $\alpha_p \in \mathbb{Z}_p = \{x \in \mathbb{Q}_p : |x|_p \leq 1\}$. \mathbb{Z}_p is the ring of p -adic integers, which they have $\nu \geq 0$ in (1). The set $\mathbb{A}_{\mathbb{Q}}$ of all adeles can be presented as

$$\mathbb{A}_{\mathbb{Q}} = \bigcup_{\mathcal{P}} A(\mathcal{P}), \quad A(\mathcal{P}) = \mathbb{R} \times \prod_{p \in \mathcal{P}} \mathbb{Q}_p \times \prod_{p \notin \mathcal{P}} \mathbb{Z}_p. \quad (3)$$

Elements of $\mathbb{A}_{\mathbb{Q}}$ satisfy componentwise addition and multiplication and form the adèle ring. Adeles and their functions are useful for connection of real and p -adic models of the same system, see, e.g. [3, 4, 5, 10, 18].

In the sequel we shall consider p -adic strings and p -adic structure of the genetic code.

3.1 p -Adic Strings

p -Adic strings are introduced by construction of their scattering amplitudes in analogy with ordinary strings [2, 14]. The simplest amplitude is for scattering of two open scalar strings. The crossing symmetric Veneziano amplitude for ordinary strings is

$$A_{\infty}(a, b) = g_{\infty}^2 \int_{\mathbb{R}} |x|_{\infty}^{a-1} |1-x|_{\infty}^{b-1} d_{\infty}x = g_{\infty}^2 \frac{\zeta(1-a)}{\zeta(a)} \frac{\zeta(1-b)}{\zeta(b)} \frac{\zeta(1-c)}{\zeta(c)}, \quad (4)$$

where $a + b + c = 1$. The crossing symmetric Veneziano amplitude for scattering of two open scalar p -adic strings is direct analog of (4) [14], i. e.

$$A_p(a, b) = g_p^2 \int_{\mathbb{Q}_p} |x|_p^{a-1} |1-x|_p^{b-1} d_p x = g_p^2 \frac{1-p^{a-1}}{1-p^{-a}} \frac{1-p^{b-1}}{1-p^{-b}} \frac{1-p^{c-1}}{1-p^{-c}}. \quad (5)$$

Integral expressions in (4) and (5) are the Gel'fand-Graev-Tate beta functions on \mathbb{R} and \mathbb{Q}_p , respectively [11]. Note that ordinary and p -adic strings differ only in description of their world-sheets – world-sheet of p -adic strings is presented by p -adic numbers. Kinematical variables contained in a, b, c are the same real (complex) numbers in both cases. Veneziano amplitude for p -adic strings (5) is rather simple and presented by elementary functions.

The above Veneziano string amplitudes are connected by the Freund-Witten product formula [15]:

$$A(a, b) = A_{\infty}(a, b) \prod_p A_p(a, b) = g_{\infty}^2 \prod_p g_p^2 = const. \quad (6)$$

Formula (6) follows from the Euler product formula for the Riemann zeta function applied to p -adic string amplitudes (5). Main significance of (6) is in the fact that scattering amplitude for real string $A_{\infty}(a, b)$, which is a special function, can be presented as product of inverse p -adic amplitudes, which are elementary functions. Also, this product formula treats p -adic and ordinary strings at the equal footing. It gives rise to suppose that if there exists an ordinary scalar string then it should exist also its p -adic analog.

It is remarkable that there is an effective field theory description of the above open p -adic strings. The corresponding Lagrangian is very simple and at the tree level describes not only four-point scattering amplitude but also all higher ones. The exact form of this Lagrangian for effective scalar field φ , which describes open p -adic string tachyon, is [16, 17]

$$\mathcal{L}_p = \frac{m^D}{g^2} \frac{p^2}{p-1} \left[-\frac{1}{2} \varphi p^{-\frac{\square}{2m^2}} \varphi + \frac{1}{p+1} \varphi^{p+1} \right], \quad (7)$$

where p is any prime number, D – space-time dimensionality, $\square = -\partial_t^2 + \nabla^2$ is the D -dimensional d'Alembertian and metric has signature $(- + \dots +)$. This is nonlocal and nonlinear Lagrangian. Nonlocality is in the form of infinite number of space-time derivatives

$$p^{-\frac{\square}{2m^2}} = \exp \left(-\frac{\ln p}{2m^2} \square \right) = \sum_{k \geq 0} \left(-\frac{\ln p}{2m^2} \right)^k \frac{1}{k!} \square^k \quad (8)$$

and it is a consequence of strings as extended objects.

It is worth noting that Lagrangian (7) can be rewritten in the form [18]

$$\mathcal{L}_p = \frac{m^D}{g^2} \frac{p^2}{p-1} \left[\frac{1}{2} \varphi \int_{\mathbb{R}} \left(\int_{\mathbb{Q}_p \setminus \mathbb{Z}_p} \chi_p(u) |u|_p^{\frac{k^2}{2m^2}} du \right) \tilde{\varphi}(k) \chi_{\infty}(kx) d^4 k + \frac{1}{p+1} \varphi^{p+1} \right], \quad (9)$$

where $\chi_{\infty}(kx) = e^{-2\pi i kx}$ is the real additive character. Since $\int_{\mathbb{Q}_p} \chi_p(u) |u|_p^{s-1} du = \frac{1-p^{s-1}}{1-p^{-s}} = \Gamma_p(s)$ and it is present in the scattering amplitude (5), one can think that expression $\int_{\mathbb{Q}_p \setminus \mathbb{Z}_p} \chi_p(u) |u|_p^{\frac{k^2}{2m^2}} du$ in (9) is related to the p -adic string world-sheet.

Now suppose that Lagrangian (7), which is written only in terms of real numbers, and its scattering amplitude

$$A_p(a, b) = g_p^2 \frac{1-p^{a-1}}{1-p^{-a}} \frac{1-p^{b-1}}{1-p^{-b}} \frac{1-p^{c-1}}{1-p^{-c}} \quad (10)$$

were discovered first and after that it was found integral representation (5) to scattering amplitudes. Then it would be natural to conclude that Lagrangian (7) describes quite new strings which have p -adic world-sheet and can be presented in the equivalent form (9). By this way, with help of p -adic numbers we are able to get some more profound information about structure of the system which is not accessible to direct measuring. For some other examples of physical models, which also include both p -adic valued and real (complex) valued functions of p -adic argument, we refer to [5] and references therein.

3.2 p -Adic Structure of the Genetic Code

The genetic code is connection between 64 codons, which are building blocks of the genes, and 20 amino acids, which are building blocks of the proteins. In addition to coding amino acids, a few codons code stop signal, which is at the end of genes and terminates process of protein synthesis. Codons are ordered triples composed of C, A, U (T) and G nucleotides. Each codon presents an information which controls use of one of the 20 standard amino acids or stop signal in synthesis of proteins. It is obvious that there are $4 \times 4 \times 4 = 64$ codons. For molecular biology and the genetic code, one can see, e.g. [19].

From mathematical point of view, the genetic code is a mapping of a set of 64 elements onto a set of 21 elements. There is in principle a huge number of possible mappings, but the genetic code is one definite mapping with a few slight modifications. Hence the main problem is to find structure of 64 elements which is used in mapping that corresponds to the genetic code. It will be demonstrated here that the set of 64 codons has p -adic structure, where $p = 5$ and 2. Detail exposition of p -adic approach to the genetic code is presented in [20, 21, 22] (see also [23] for a similar consideration).

The idea behind this approach is as follows. Codons which code the same amino acid should be in information sense close. To quantify this closeness (nearness) we should use some distance. Ordinary distance is appropriate to determine spatial distribution of codons but not for their distribution with respect to information characteristics. From insight to the table of the genetic code (see, e. g. Table 1) one can conclude that distribution of codons is like an ultrametric tree and it suggests to use p -adic distance.

To this end, let us introduce the following subset of natural numbers:

$$\mathcal{C}_5[64] = \{n_0 + n_1 5 + n_2 5^2 : n_i = 1, 2, 3, 4\}, \quad (11)$$

where n_i are digits different from zero. This is a finite expansion to the base 5. It is obvious that 5 is a prime number and that the set $\mathcal{C}_5[64]$ contains 64 natural numbers. It is convenient to denote elements of $\mathcal{C}_5[64]$ by their digits to the base 5 in the following way: $n_0 + n_1 5 + n_2 5^2 \equiv n_0 n_1 n_2$. Here ordering of digits is the same as in the expansion and it is opposite to the usual one.

Now we are interested in 5-adic distances between elements of $\mathcal{C}_5[64]$. It is worth recalling p -adic norm between integers, which is related to the divisibility of integers by prime numbers. Difference of two integers is again an integer. p -Adic distance between two integers can be understood as a measure of divisibility of their difference by p (the more divisible, the shorter). By definition, p -adic norm of an integer

$m \in \mathbb{Z}$, is $|m|_p = p^{-k}$, where $k \in \mathbb{N} \cup \{0\}$ is degree of divisibility of m by prime p (i.e. $m = p^k m'$, $p \nmid m'$) and $|0|_p = 0$. This norm is a mapping from \mathbb{Z} into non-negative rational numbers. One can easily conclude that $0 \leq |m|_p \leq 1$ for any $m \in \mathbb{Z}$ and any prime p .

p -Adic distance between two integers x and y is

$$d_p(x, y) = |x - y|_p. \quad (12)$$

Since p -adic absolute value is ultrametric, the p -adic distance (12) is also ultrametric, i.e. it satisfies inequality

$$d_p(x, y) \leq \max \{d_p(x, z), d_p(z, y)\} \leq d_p(x, z) + d_p(z, y), \quad (13)$$

where x, y and z are any three integers.

5-Adic distance between two numbers $a, b \in \mathcal{C}_5 [64]$ is

$$d_5(a, b) = |a_0 + a_1 5 + a_2 5^2 - b_0 - b_1 5 - b_2 5^2|_5, \quad (14)$$

where $a_i, b_i \in \{1, 2, 3, 4\}$. When $a \neq b$ then $d_5(a, b)$ may have three different values:

- $d_5(a, b) = 1$ if $a_0 \neq b_0$,
- $d_5(a, b) = 1/5$ if $a_0 = b_0$ and $a_1 \neq b_1$,
- $d_5(a, b) = 1/5^2$ if $a_0 = b_0$, $a_1 = b_1$ and $a_2 \neq b_2$.

We see that the largest 5-adic distance between numbers is 1 and it is maximum p -adic distance on \mathbb{Z} . The smallest 5-adic distance on the space $\mathcal{C}_5 [64]$ is 5^{-2} . Note that 5-adic distance depends only on the first two digits of different numbers $a, b \in \mathcal{C}_5 [64]$.

Ultrametric space $\mathcal{C}_5 [64]$ can be viewed as 16 quadruplets with respect to the smallest 5-adic distance, i.e. quadruplets contain 4 elements and 5-adic distance between any two elements within quadruplet is $\frac{1}{25}$. In other words, within each quadruplet elements have the first two digits equal and third digits are different.

With respect to 2-adic distance, the above quadruplets may be viewed as composed of two doublets: $a = a_0 a_1 1$ and $b = a_0 a_1 3$ make the first doublet, and $c = a_0 a_1 2$ and $d = a_0 a_1 4$ form the second one. 2-Adic distance between codons within each of these doublets is $\frac{1}{2}$, i.e.

$$d_2(a, b) = |(3 - 1) 5^2|_2 = \frac{1}{2}, \quad d_2(c, d) = |(4 - 2) 5^2|_2 = \frac{1}{2}. \quad (15)$$

By this way ultrametric space $\mathcal{C}_5[64]$ of 64 elements is arranged into 32 doublets.

Identifying appropriately nucleotides by digits, we obtain the corresponding ultrametric structure of the codon space in the vertebrate mitochondrial genetic code. We take the following assignments between nucleotides and digits in $\mathcal{C}_5[64]$: C (cytosine) = 1, A (adenine) = 2, T (thymine) = U (uracil) = 3, G (guanine) = 4. There is now evident one-to-one correspondence between codons in three-letter notation and three-digit $n_0 n_1 n_2$ number representation of ultrametric space $\mathcal{C}_5[64]$, see Table 1.

The above introduced set $\mathcal{C}_5[64]$ endowed by p -adic distance we call *5-adic codon space* (or *5-adic space of codon states*), because elements of $\mathcal{C}_5[64]$ represent codons (or codon states) denoted by $n_0 n_1 n_2$.

Let us now explore distances between codons. To this end, it is useful to look at the Table 1 as a representation of the $\mathcal{C}_5[64]$ codon space. We observe that there are 16 quadruplets such that each of them has the same first two digits. Hence 5-adic distance between any two different codons within a quadruplet is

$$\begin{aligned} d_5(a, b) &= |a_0 + a_1 5 + a_2 5^2 - a_0 - a_1 5 - b_2 5^2|_5 \\ &= |(a_2 - b_2) 5^2|_5 = |(a_2 - b_2)|_5 |5^2|_5 = 5^{-2}, \end{aligned} \quad (16)$$

because $a_0 = b_0$, $a_1 = b_1$ and $|a_2 - b_2|_5 = 1$. According to (16) codons within every quadruplet are at the smallest 5-adic distance, i.e. they are closest comparing to all other codons.

Since codons are composed of three ordered nucleotides, each of them is either purine or pyrimidine, it is desirable to quantify nearness inside purines and pyrimidines, as well as distinction between elements from these two groups of nucleotides. This is natural to do by 2-adic distance. Namely, one can easily see that 2-adic distance between pyrimidines C and U is $d_2(1, 3) = |3 - 1|_2 = 1/2$ as the distance between purines A and G is $d_2(2, 4) = |4 - 2|_2 = 1/2$. However 2-adic distance between C and A or G as well as distance between U and A or G is 1 (i.e. maximum).

By the above application of 5-adic and 2-adic distances to $\mathcal{C}_5[64]$ codon space we have obtained internal structure of the codon space in the form of doublets. Just this p -adic structure of codon space with doublets corresponds to the mapping which we find in the vertebrate mitochondrial genetic code. The other (about 20) known versions of the genetic code in living systems can be viewed as slight modifications of this mitochondrial code, presented at the Table 1.

111	CCC	Pro	211	ACC	Thr	311	UCC	Ser	411	GCC	Ala
112	CCA	Pro	212	ACA	Thr	312	UCA	Ser	412	GCA	Ala
113	CCU	Pro	213	ACU	Thr	313	UCU	Ser	413	GCU	Ala
114	CCG	Pro	214	ACG	Thr	314	UCG	Ser	414	GCG	Ala
121	CAC	His	221	AAC	Asn	321	UAC	Tyr	421	GAC	Asp
122	CAA	Gln	222	AAA	Lys	322	UAA	Ter	422	GAA	Glu
123	CAU	His	223	AAU	Asn	323	UAU	Tyr	423	GAU	Asp
124	CAG	Gln	224	AAG	Lys	324	UAG	Ter	424	GAG	Glu
131	CUC	Leu	231	AUC	Ile	331	UUC	Phe	431	GUC	Val
132	CUA	Leu	232	AUA	Met	332	UUA	Leu	432	GUA	Val
133	CUU	Leu	233	AUU	Ile	333	UUU	Phe	433	GUU	Val
134	CUG	Leu	234	AUG	Met	334	UUG	Leu	434	GUG	Val
141	CGC	Arg	241	AGC	Ser	341	UGC	Cys	441	GGC	Gly
142	CGA	Arg	242	AGA	Ter	342	UGA	Trp	442	GGA	Gly
143	CGU	Arg	243	AGU	Ser	343	UGU	Cys	443	GGU	Gly
144	CGG	Arg	244	AGG	Ter	344	UGG	Trp	444	GGG	Gly

Table 1. The p -adic vertebrate mitochondrial genetic code.

4 Conclusion

In this paper I have discussed some aspects of measurements, p -adic numbers and their applications in mathematical physics and related topics. In particular, I used p -adic strings and the genetic code to demonstrate how p -adic modelling works and can be useful.

It is also emphasized that results of measurements are rational numbers with Archimedean norm and natural ordering. Thus, p -adic numbers are not results of measurements, but nevertheless they play important role in p -adic mathematical physics and its applications. In particular, one can expect their further significant role in description of information sector of the living systems (see, e. g. [24]).

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